

# Multiplex Heterogeneous Graph Convolutional Network

Pengyang Yu  
Ocean University of China  
Qingdao, China  
ypy@stu.ouc.edu.cn

Chaofan Fu  
Ocean University of China  
Qingdao, China  
fuchaofan@stu.ouc.edu.cn

Yanwei Yu\*  
Ocean University of China  
Qingdao, China  
yuyanwei@ouc.edu.cn

Chao Huang\*  
The University of Hong Kong  
Hong Kong, China  
chaohuang75@gmail.com

Zhongying Zhao  
Shandong University of Science and  
Technology  
Qingdao, China  
zyzhao@sdust.edu.cn

Junyu Dong  
Ocean University of China  
Qingdao, China  
dongjunyu@ouc.edu.cn

## ABSTRACT

Heterogeneous graph convolutional networks have gained great popularity in tackling various network analytical tasks on heterogeneous network data, ranging from link prediction to node classification. However, most existing works ignore the relation heterogeneity with multiplex network between multi-typed nodes and different importance of relations in meta-paths for node embedding, which can hardly capture the heterogeneous structure signals across different relations. To tackle this challenge, this work proposes a **Multiplex Heterogeneous Graph Convolutional Network (MHGCN)** for heterogeneous network embedding. Our MHGCN can automatically learn the useful heterogeneous meta-path interactions of different lengths in multiplex heterogeneous networks through multi-layer convolution aggregation. Additionally, we effectively integrate both multi-relation structural signals and attribute semantics into the learned node embeddings with both unsupervised and semi-supervised learning paradigms. Extensive experiments on five real-world datasets with various network analytical tasks demonstrate the significant superiority of MHGCN against state-of-the-art embedding baselines in terms of all evaluation metrics. The source code of our method is available at: <https://github.com/NSSJSS/MHGCN>.

## CCS CONCEPTS

• **Mathematics of computing** → **Graph algorithms**; • **Computing methodologies** → **Learning latent representations**.

## KEYWORDS

Network Embedding; Graph Representation Learning; Multiplex Heterogeneous Networks; Graph Convolutional Networks

\*Yanwei Yu and Chao Huang are the corresponding authors.

Permission to make digital or hard copies of all or part of this work for personal or classroom use is granted without fee provided that copies are not made or distributed for profit or commercial advantage and that copies bear this notice and the full citation on the first page. Copyrights for components of this work owned by others than ACM must be honored. Abstracting with credit is permitted. To copy otherwise, or republish, to post on servers or to redistribute to lists, requires prior specific permission and/or a fee. Request permissions from [permissions@acm.org](mailto:permissions@acm.org).  
*KDD '22, August 14–18, 2022, Washington, DC, USA*

© 2022 Association for Computing Machinery.  
ACM ISBN 978-1-4503-9385-0/22/08...\$15.00  
<https://doi.org/10.1145/3534678.3539482>

## ACM Reference Format:

Pengyang Yu, Chaofan Fu, Yanwei Yu, Chao Huang, Zhongying Zhao, and Junyu Dong. 2022. Multiplex Heterogeneous Graph Convolutional Network. In *Proceedings of the 28th ACM SIGKDD Conference on Knowledge Discovery and Data Mining (KDD'22)*, August 14–18, 2022, Washington, DC, USA. ACM, New York, NY, USA, 11 pages. <https://doi.org/10.1145/3534678.3539482>

## 1 INTRODUCTION

Network representation learning has emerged as a new learning paradigm to embed complex network into a low-dimensional vector space while preserving the proximities of nodes in both network topological structures and intrinsic properties. Effective network representation advances various network analytical tasks, ranging from link prediction [3, 19, 24], node classification [6, 20, 34], to recommendation [13, 14, 27]. In recent years, Graph Convolutional Networks (GCNs) [16], a class of neural networks designed to learn graph representation for complex networks with rich feature information, have been applied to many online services, such as E-commerce [17], social media platforms [35] and advertising [8].

While many efforts have been made to study the representation learning over homogeneous graphs [6, 16, 25, 28], the exploration of preserving network heterogeneous properties in graph representation paradigms has attracted much attention in recent studies, e.g., *metapath2vec* [4] and *HERec* [27]. Inspired by the strength of Graph Neural Networks (GNNs) in aggregating contextual signals from neighboring nodes, various graph neural models have been introduced to tackle the challenge of heterogeneous graph learning, such as *HAN* [30], *MAGNN* [5] and *HetGNN* [40].

Albeit the effectiveness of existing heterogeneous network embedding methods [4, 14, 22, 31], these works are generally designed for homogeneous networks with a single view. In real-world scenarios, however, many networks are much more complex, comprising not only multi-typed nodes and diverse edges even between the same pair-wise nodes but also a rich set of attributes [1]. For example, in E-commerce networks, there are two types of nodes (i.e., users and items), and multiple relations (e.g., click, purchase, add-to-cart, or add-to-preference) between the same pairs of users and items [36]. The connections between multiple types of nodes in such networks are often heterogeneous with relation diversity, which yields networks with multiple different views. It is worth noting that the multiplicity of the network is fundamentally different from the heterogeneity of the network. Two types of nodes, users and items, in a E-commerce network reflect the heterogeneity

of the network. At the same time, users may have several types of interactions (*e.g.*, click, purchase, review) with items [33], which reflects the multiplex relationships of the network. Because different user-item interactions exhibit different views of user and item, and thus should be treated differently. We term this kind of networks with both multiplex network structures with multi-typed nodes and node attribute information as **attributed multiplex heterogeneous networks** (AMHENS).

Performing representation learning on the AMHENS is of great importance to network mining tasks, yet it is very challenging due to such complicated network structures and node attributes. While some recent studies propose to solve the representation learning problem on multiplex heterogeneous network [1, 20, 23, 37, 39], several key limitations exist in those methods. i) The success of current representation learning models largely relies on the accurate design of meta-paths. How to design an automated learning framework to explore the complex meta-path-based dependencies over the multiplex heterogeneous graphs, remains a significant challenge. ii) Unlike the homogeneous node aggregation scheme, with the heterogeneous node types and multiplex node relationships, each meta-path can be regarded as relational information channel. An effective meta-path dependency encoder is a necessity to inject both the relation heterogeneity and multiplexity into the representations. iii) In real-world graph representation scenarios, efficiency is an important factor to handle the graph data with large number of heterogeneous nodes and multiplex edges. However, most current methods are limited to serve the large-scale network data, due to their high time complexity and memory consumption.

To address the aforementioned challenges, we propose a new **Multiplex Heterogeneous Graph Convolutional Network**, named **MHGCN**, for AMHEN embedding. Specifically, we first decouple the multiplex network into multiple homogeneous and bipartite sub-networks, and then re-aggregate the sub-networks with the exploration of their importance (*i.e.*, weights) in node representation learning. To automatically capture meta-path information across multi-relations, we tactfully design a multilayer graph convolution module, which can effectively learn the useful heterogeneous meta-path interactions of different lengths in AMHENS through multilayer convolution aggregation in both unsupervised and semi-supervised learning paradigms. To improve the model efficiency, we endow our MHGCN with a simplified graph convolution for feature aggregation, in order to significantly reduce the model computational cost. Our evaluations are conducted on several real-world graph datasets to evaluate the model performance in both link prediction and node classification tasks. Experimental results show that our MHGCN framework can obtain the substantial performance improvement compared with state-of-the-art graph representation techniques. With the designed graph convolution module, our MHGCN achieves better model efficiency when competing with state-of-the-art GNN baselines for AMHENS by up to two orders of magnitudes (see efficiency analysis in the supplemental material).

We summarize the contributions of this paper as follows:

- We propose an effective multiplex heterogeneous graph neural network, MHGCN, which can automatically capture the useful relation-aware topological structural signals between

nodes for heterogeneous network embedding.

- MHGCN integrates both network structures and node attribute features in node representations, and gains the capability to efficiently learn network representation with a simplified convolution-based message passing mechanism.
- We conduct extensive experiments on five real-world datasets to verify the superiority of our proposed model in both link prediction and node classification when competing with state-of-the-art baselines.

## 2 RELATED WORK

**Graph Neural Networks.** The goal of a GNN is to learn a low-dimensional vector representation for each node, which can be used for many downstream network mining tasks. Kipf *et al.* [16] proposes to perform convolutional operations over graph neighboring node for information aggregation. GraphSAGE [7] is an inductive GNN framework, which uses the general aggregating functions for efficient generation of node embeddings. To differentiate the influence of neighboring nodes, GAT [29] has been proposed as an attentive message passing mechanism to learn the explicit weights of neighbor node embeddings. R-GCN [26] considers the influence of different edge types on nodes, and uses weight sharing and coefficient constraints to apply to multi-graphs with large numbers of relations. To simplify the design of graph convolutional network, LightGCN [9] omits the embedding projection with non-linearity during the message passing. Additionally, AM-GCN [32] is proposed to adaptively learn deep correlation information between topological structures and node features. However, all algorithms mentioned above are developed for the homogeneous networks, and thus cannot effectively preserve the heterogeneous and multiplex graph characteristics for the network representation task.

**Heterogeneous Graph Representation.** Modeling the heterogeneous context of graphs has already received some attention [4, 12, 21, 27, 40]. For example, some studies leverage random walks to construct meta-paths over the heterogeneous graph for node embeddings, including *metapath2vec* [4] and *HERec* [27]. As graph neural networks (GNNs) have become a popular choice for encoding graph structures, many heterogeneous graph neural network models are designed to enhance the GNN architecture with the capability of capturing the node and edge heterogeneous contextual signals. For example, *HetGNN* [40] jointly encodes the graph topology and context heterogeneity for representation learning. *HeGAN* [10] incorporates generative adversarial networks (GAN) for heterogeneous network embedding. *NARS* [38] first generates relation subgraphs, learns node embeddings by 1D convolution on the subgraphs and then aggregates the learned embeddings. *Fu et al.* [5] performs both the intra- and inter-metapath aggregation so as to distill the metapath-based relational context for learning node representations. However, most of those approaches rely on selecting the useful metapaths to guide the process of heterogeneous representation, which may need the external domain knowledge for constructing relevant metapaths.

In addition, there exist some recent studies attempting to relax the requirement of metapath construction for heterogeneous graph

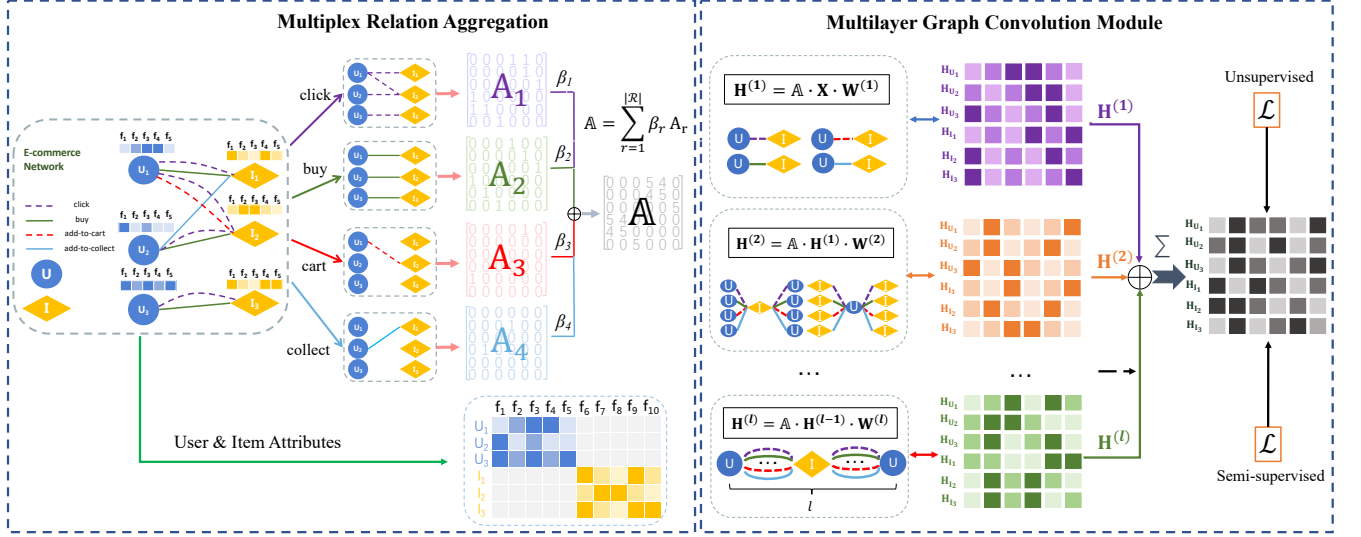


Figure 1: The overview of the proposed MHGCN.

representations. In particular, HGT [11] proposes to incorporate the self-attention into the graph-based message passing mechanism for modeling the dynamic dependencies among heterogeneous nodes. HPN [14] eliminates semantic confusion by mapping nodes in meta-path to semantic space, and then aggregates the embeddings of nodes under different metapaths to obtain the final representation. However, most of the above heterogeneous graph embedding models ignore the multiplex relational context of real-life graph data, in which multi-typed relationships exist among nodes.

**Multiplex Heterogeneous Network Embedding.** Real-world graphs are often inherently multiplex, which involves various relations and interactions between two connected nodes. To tackle this challenge, many multiplex network embedding techniques are proposed to project diverse node edges into latent representations. For example, MNE [41] introduces a global transformation matrix for each layer of the network to align the embeddings with different dimensions for each relation type. GATNE [1] splits the node representation by learning base embedding, edge embedding as well as attribute embedding. The self-attention is utilized to fuse neighborhood information for generating edge representation. Motivated by the mutual information maximization scheme, DMGI [23] is proposed as an unsupervised learning approach which aims to minimize the difference among relation-aware node representations. HGSL [43] first obtains the node representation based on meta-paths, and then uses GNN to jointly train the heterogeneous graph, node representation and node attributes to obtain the final embedding. However, the generality of the above methods is limited by their manual construction of meta-paths.

Recently, FAME [20] develops a spectral graph transformation component to aggregate information from sub-networks by preserving relation-aware node dependencies. However, this model is built on the random projection and sacrifices the adaptive parameter learning in exchange for fast embedding projection. Furthermore, to learn the node embeddings of multiplex bipartite graph, Dual-HGCN [37] firstly generates two sets of homogeneous hypergraphs

and then perform the information propagation with the spectral hypergraph convolutions. In HDI [15], Jing *et al.* explores the high-order mutual information to construct the supervision signals for enhancing the node representations.

### 3 PROBLEM DEFINITION

We define graph  $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$  with the set of nodes  $\mathcal{V}$  and edges  $\mathcal{E}$ . Each edge in  $\mathcal{E}$  represents the connections among nodes.

**DEFINITION 1 (ATTRIBUTED MULTIPLEX HETEROGENEOUS NETWORK, OR AMHEN).** Given the defined graph  $\mathcal{G}$ , we further associate all nodes in  $\mathcal{V}$  with the attribute feature vectors  $\mathbf{X} \in \mathbb{R}^{n \times m}$ . Here, the size of node set  $\mathcal{V}$  and attribute vector is represented by  $n$  and  $m$ , respectively. With the consideration of node and edge heterogeneity, we define the node type and edge type mapping function as  $\phi: \mathcal{V} \rightarrow \mathcal{O}$  and  $\psi: \mathcal{E} \rightarrow \mathcal{R}$ . Here, the set of node types and edge types is set with the size of  $\mathcal{O}$  and  $\mathcal{R}$ , respectively. Each node  $v \in \mathcal{V}$  and edge  $e \in \mathcal{E}$  are associated with a specific type in  $\mathcal{O}$  and  $\mathcal{R}$ , respectively. Additionally, with the consideration of edge multiplicity (i.e.,  $|\mathcal{O}| + |\mathcal{R}| > 2$ ), the same pair of nodes can be connected through multi-typed edges.

**DEFINITION 2 (META-PATH).** A meta-path  $\mathcal{P}$  is defined as a path in the form of  $O_1 \xrightarrow{r_1} O_2 \xrightarrow{r_2} \dots \xrightarrow{r_{l-1}} O_l$  which describes a composite relation  $R = r_1 \circ r_2 \circ \dots \circ r_{l-1}$  between node types  $O_1$  and  $O_l$ , where  $\circ$  denotes the composition operator on relations.

For example,  $U_1 \xrightarrow{\text{click}} I_2 \xrightarrow{\text{buy}} U_2$  is a meta-path sample of meta-path  $User \xrightarrow{\text{click}} Item \xrightarrow{\text{buy}} User$ . Based on the above definitions, we formally present the representation learning task over the multiplex heterogeneous graph as follows:

**PROBLEM (ATTRIBUTED MULTIPLEX HETEROGENEOUS GRAPH REPRESENTATION).** The objective of our representation learning task over the attributed multiplex heterogeneous graph  $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathbf{X}\}$  is to learn low-dimensional latent embedding (with the hidden dimensionality of  $d \ll |\mathcal{V}|$ ) for each node  $v \in \mathcal{V}$ , with the preservation of node and edge heterogeneity and multiplicity.

We summarize the key notations of our technical solution in Table 4 presented in the supplementary material.

## 4 METHODOLOGY

This section describe our framework MHGCN with the overall architecture shown in Figure 1. Particularly, our MHGCN contains two key learning modules: (i) *multiplex relation aggregation* and (ii) *multilayer graph convolution module*. *Multiplex relation aggregation* aims to aggregate the multi-relations among heterogeneous nodes in multiplex heterogeneous networks by differentiating each relation with importance. *Multilayer graph convolution module* can automatically capture the heterogeneous meta-paths of different lengths across multi-relations by aggregating neighboring nodes' characteristics to learn the low-dimensional representation of nodes.

### 4.1 Multiplex Relation Aggregation

As defined in Sec. 3, there exist different types of nodes and multiple types of edges between these nodes in AMHENS, and each type of edge has a different role and impact on node representation. Therefore, following [20], we first generate multiple sub-graphs by differentiating the types of edge connections between nodes in the multiplex and heterogeneous graph. Afterwards, we aggregate the relation-aware graph contextual information with different importance weights.

We denote our generated sub-graph as  $\{\mathcal{G}_r | r = 1, 2, \dots, |\mathcal{R}|\}$  with the corresponding adjacent matrix  $\{\mathbf{A}_r | r = 1, 2, \dots, |\mathcal{R}|\}$ . Considering the scenario of multiplex user-item relations in online retailer (e.g., click, purchase, review), the decomposed sub-graph corresponds to individual type of relationship between user and item. For instance, for the graph representation learning in E-commerce platforms, different relationships (different edge types) between user and item nodes exhibit various dependency semantics. For example, the diverse behaviors of users (e.g., click, add-to-favorite, purchase) reflect different preferences of users over items. Hence, multiplex user-item interactions with various relation semantics will have different impacts on the learning process of user representations. To capture such multi-typed node dependencies, our proposed MHGCN learns the relation-aware weights  $\beta_r$  to aggregate edge-type-specific sub-graph adjacent matrix as:  $\mathbb{A} = \sum_{r=1}^{|\mathcal{R}|} \beta_r \mathbf{A}_r$ . Notice that the set of weights  $\{\beta_r | r = 1, 2, \dots, |\mathcal{R}|\}$  should not be a set of hyperparameter, but should be dynamically changed according to different tasks, so we set them as trainable parameters to be learned in model training.

### 4.2 Multilayer Graph Convolution Module

Different from homogeneous networks, heterogeneous networks contain different types of nodes and edges. The specified types of edges and nodes form a meta-path, which has an obvious effect on the representation learning of heterogeneous networks. Previous works require manually defined meta-paths and learn node representations on the sampled heterogeneous meta-paths. However, setting and sampling meta-paths artificially is a complex task. In a large-scale network, the number of meta-paths is very large. It takes a long time to sample such a large number of meta-paths. At the same time, aggregating meta-paths into meta-path graph also requires a lot of memory overhead. Additionally, the type of

meta-paths has an important impact on node representation, which almost determines the performance of network embedding in various downstream tasks. The number of types of heterogeneous meta-paths is also very large, involving different lengths and different relation interactions. Therefore, it is difficult to select the appropriate meta-path types for heterogeneous network embedding methods based on meta-path aggregation. Our MHGCN effectively solves the above problems. We now present our multilayer graph convolution module that automatically captures the the short and long meta-paths across multi-relations in AMHENS.

It is worth noting that our model uses a multi-layer fusion GCN. As shown in Figure 1, our graph convolution module consists of multiple graph convolutional layers. Its purpose is to capture meta-path information of different lengths. Next, we take a two-layer GCN as an example to illustrate how our model capture meta-path information. For a single layer GCN:

$$\mathbf{H}^{(1)} = \mathbb{A} \cdot \mathbf{X} \cdot \mathbf{W}^{(1)}, \quad (1)$$

where  $\mathbf{H}^{(1)} \in \mathbb{R}^{n \times d}$  is the output of first layer (i.e., hidden representation of network),  $\mathbf{X} \in \mathbb{R}^{n \times m}$  is the node attribute matrix, and  $\mathbf{W}^{(1)} \in \mathbb{R}^{m \times d}$  is the learnable weight matrix. Notice that our convolution adopts the idea of simplifying GCN [34], that is, no non-linear activation function is used.

For the two-layer GCN, the message passing process can be represented as below:

$$\begin{aligned} \mathbf{H}^{(2)} &= \mathbb{A} \cdot \mathbf{H}^{(1)} \cdot \mathbf{W}^{(2)} \\ &= \mathbb{A} \cdot (\mathbb{A} \cdot \mathbf{X} \cdot \mathbf{W}^{(1)}) \cdot \mathbf{W}^{(2)} \\ &= \mathbb{A}^2 \cdot \mathbf{X} \cdot \mathbf{W}^{(1)} \cdot \mathbf{W}^{(2)}, \end{aligned} \quad (2)$$

where  $\mathbf{W}^{(2)} \in \mathbb{R}^{d \times d}$  is the learnable weight matrix for second layer.

We present an illustrated example with a graph generated from E-commerce data in Figure 2 based on two types of node relations, namely users' buy and click behaviors on items. As shown in Figure 2, aggregated matrix  $\mathbb{A}$  can be regarded as a meta-path graph matrix generated by the 1-length meta-paths with importance (i.e., all linked node pairs across all edge types with weights). For example,  $\mathbb{A}_{(1,3)} = 1.5$  contains two 1-length meta-path sam-

ples with weights, i.e.,  $U_1 \xrightarrow{1*buy} I_1 : 1$  and  $U_1 \xrightarrow{0.5*click} I_1 : 0.5$ . Therefore, the single-layer GCN can effectively learn the node representation that contains 1-length meta-path information. Similarly, the second power of  $\mathbb{A}$  automatically captures the 2-length meta-path information with importance weights for all node pairs, including original sub-network high-order structures. For example,  $\mathbb{A}_{(1,1)}^2 = 2.5$  implies five 2-length meta-path samples across multi-relations with importance, i.e.,  $U_1 \xrightarrow{1*buy} I_1 \xrightarrow{1*buy} U_1 : 1$ ,  $U_1 \xrightarrow{1*buy} I_1 \xrightarrow{0.5*click} U_1 : 0.5$ ,  $U_1 \xrightarrow{0.5*click} I_1 \xrightarrow{0.5*click} U_1 : 0.25$ ,  $U_1 \xrightarrow{0.5*click} I_1 \xrightarrow{1*buy} U_1 : 0.5$ , and  $U_1 \xrightarrow{0.5*click} I_2 \xrightarrow{0.5*click} U_1 : 0.25$ . The sum of the importance of these five meta-path samples is 2.5.

At the same time, considering that the influence of meta-paths with different lengths on embedding should also be different, the learnable weight matrices  $\mathbf{W}^{(l)}$  in our multilayer graph convolution module can just play this role. Eventually, we fuse the outputs of

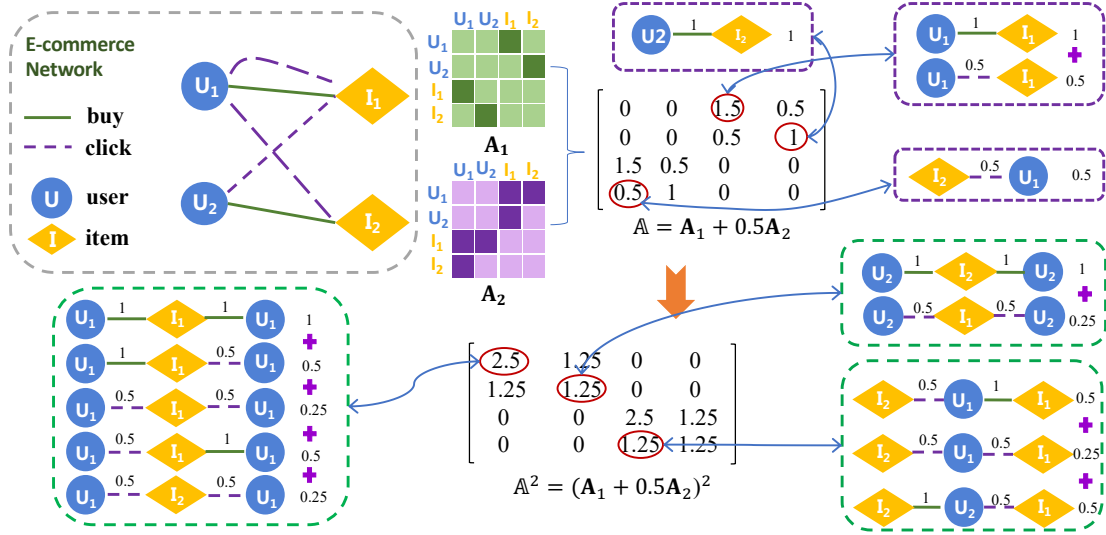


Figure 2: Illustration of meta-paths with importance for a toy example

single-layer GCN and two-layer GCN:

$$\mathbf{H} = \frac{1}{2}(\mathbf{H}^{(1)} + \mathbf{H}^{(2)}). \quad (3)$$

The final embedding  $\mathbf{H} \in \mathbb{R}^{n \times d}$  contains all 1-length and 2-length meta-path information.

To capture the more length heterogeneous meta-paths, we can extend it to  $l$ -layer:

$$\begin{aligned} \mathbf{H}^{(l)} &= \mathbb{A} \cdot \mathbf{H}^{(l-1)} \cdot \mathbf{W}^{(l)} \\ &= \mathbb{A} \cdot (\mathbb{A} \cdot \mathbf{H}^{(l-2)} \cdot \mathbf{W}^{(l-1)}) \cdot \mathbf{W}^{(l)} \\ &= \underbrace{\mathbb{A} \cdots \mathbb{A}}_l \cdot \underbrace{\mathbf{X} \cdot \mathbf{W}^{(1)} \cdots \mathbf{W}^{(l)}}_l \\ &= \mathbb{A}^l \cdot \underbrace{\mathbf{X} \cdot \mathbf{W}^{(1)} \cdots \mathbf{W}^{(l)}}_l \end{aligned} \quad (4)$$

Therefore, our multilayer graph convolution module fuses outputs of all layers to capture all meta-path information of different length across multi-relations:

$$\begin{aligned} \mathbf{H} &= \frac{1}{l} \sum_{i=1}^l \mathbf{H}^{(i)} \\ &= \frac{1}{l} \sum_{i=1}^l \mathbb{A} \cdot \mathbf{H}^{(i-1)} \cdot \mathbf{W}^{(i)}, \end{aligned} \quad (5)$$

where  $\mathbf{H}^{(0)}$  is the node attribute matrix  $\mathbf{X}$ .

### 4.3 Model Learning

This part presents the defined objective function to train our model to learn the final node representation. Depending on the requirements of different downstream tasks and the availability of node labels, we can train MHGCN in two major learning paradigms, *i.e.*, unsupervised learning and semi-supervised learning.

For unsupervised learning, we can optimize the model parameters by minimizing the following binary cross-entropy loss function through negative sampling:

$$\mathcal{L} = - \sum_{(u,v) \in \Omega} \log \sigma(\langle \mathbf{H}_u^T, \mathbf{H}_v \rangle) - \sum_{(u',v') \in \Omega^-} \log \sigma(-\langle \mathbf{H}_{u'}^T, \mathbf{H}_{v'} \rangle), \quad (6)$$

where  $\mathbf{H}_v$  is the representation of node  $v$ ,  $\top$  denotes matrix transposition,  $\sigma(\cdot)$  is the sigmoid function,  $\langle \cdot, \cdot \rangle$  can be any vector similarity measure function (*e.g.*, inner product),  $\Omega$  is the set of positive node pairs,  $\Omega^-$  is the set of negative node pairs sampled from all unobserved node pairs. That is, we use the loss function to increase the similarities between the node representations in the positive samples and decrease the similarities between the node representations in the negative samples simultaneously.

For semi-supervised learning, we can optimize the model parameters by minimizing the cross entropy via backpropagation and gradient descent. The cross entropy loss over all labeled nodes between the ground-truth and the prediction is formulated as:

$$\mathcal{L} = - \sum_{i \in \mathcal{V}_{ids}} Y_i \ln(C \cdot \mathbf{H}_i), \quad (7)$$

where  $\mathcal{V}_{ids}$  is the set of node indices that have labels,  $Y_i$  is the label of the  $i$ -th node,  $C$  is the node classifier parameter, and  $\mathbf{H}_i$  is the representation of the  $i$ -th node. With the guide of a small fraction of labeled nodes, we can optimize the proposed model and then learn the embeddings of nodes for semi-supervised classification.

Notice that  $\{\mathbf{W}^{(i)} | i = 1, 2, \dots, l\}$  and  $\{\beta_r | r = 1, 2, \dots, |\mathcal{R}|\}$  in our model can be learned during training phase. The pseudo-code of our proposed MHGCN is shown in Algorithm 1 in the supplement.

**Table 1: Statistical information of evaluation network datasets (node type: n-type, edge type: e-type, features: feat., and Multiplex network: Mult.)**

Dataset	#nodes	#edges	#n-type	#e-type	#feat.	Mult.
Alibaba	21,318	41,676	2	4	19	✓
Amazon	10,166	148,865	1	2	1,156	✓
AMiner	58,068	118,939	3	3	4	×
IMDB	12,772	18,644	3	2	1,256	×
DBLP	26,128	119,783	4	3	4,635	×

## 5 EXPERIMENT

### 5.1 Datasets

In our evaluation, five publicly available real-world datasets are used in experimental evaluation, *i.e.*, Alibaba<sup>1</sup>, Amazon<sup>2</sup>, AMiner<sup>3</sup>, IMDB<sup>4</sup>, and DBLP<sup>5</sup>. Detailed dataset description can be found in the supplement. Due to the scalability limitation of applying some baselines in the whole Alibaba network data, we evaluate all models on a sampled dataset from Alibaba. The statistics of these five datasets are summarized in Table 1.

### 5.2 Baselines

We compare our MHGCN against the following eighteen graph learning baselines, which are divided into three categories.

Homogeneous network embedding methods:

- **node2vec** [6] - node2vec is a representative method for graph representation by leveraging the random walk to generate node sequences over graphs.
- **RandNE** [42] - RandNE performs projection process in an iterative way to capture the high-order graph structures with the matrix factorization objective.
- **FastRP** [2] - This method generates similarity matrix for modeling the transitive relations among nodes. Then, FastRP leverages sparse random projection to reduce dimension.
- **SGC** [34] - SGC proposes to simplify the graph convolutional networks by removing the non-linear projection during the information propagation between graph layers.
- **AM-GCN** [32] - AM-GCN is a state-of-the-art graph convolutional network, which is an adaptive multi-channel graph convolutional networks for semi-supervised classification.

Heterogeneous network embedding methods:

- **R-GCN** [26] - R-GCN further considers the influence of different edge types on nodes, and uses weight sharing and coefficient constraints to apply to heterogeneous networks.
- **HAN** [30] - HAN applies graph attention network on multiplex network considering the inter- and intra-network interactions, which exploit manually selected meta-paths to learn node embedding.

- **NARS** [38] - NARS decouples heterogeneous networks according to the type of edge, and then aggregates neighbor features on the decoupled subgraph.
- **MAGNN** [5] - MAGNN is a metapath aggregated graph neural network for heterogeneous graphs.
- **HPN** [14] - HPN designs a semantic propagation mechanism to alleviate semantic confusion and a semantic fusion mechanism to integrate rich semantics.

Multiplex Heterogeneous network embedding methods:

- **PMNE** [18] - PMNE contains three different models to merge the multiplex network to generate one overall embedding for each node, which are denoted as PMNE-n, PMNE-r, and PMNE-c, respectively.
- **MNE** [41] - MNE obtains the final embedding by combining the high-dimensional common embedding and the low-dimensional hierarchical embedding.
- **GATNE** [1] - GATNE proposes to generate the overall node embeddings with the base embedding as well as the edge and attribute representations. The edge embedding is generated by fusing neighboring information using self-attention.
- **GTN** [39] - It studies the graph representation task by identifying effective meta-paths with high-order relations.
- **DMGI** [23] - DMGI develops a consensus regularization scheme to consider the relations among type-specific node embeddings. Furthermore, each relational contextual signals are aggregated through attention mechanism.
- **FAME** [20] - FAME is a random projection-based network embedding for AMHENS, which uses spectral graph transformation to capture meta-paths, and significantly improves efficiency through random projection.
- **HGSL** [43] - HGSL is a state-of-the-art heterogeneous GNN, which jointly performs heterogeneous graph structure learning and GNN parameter learning for classification.
- **DualHGNN** [37] - DualHGNN uses dual hypergraph convolutional network to learn node embeddings for multiplex bipartite networks.

The network types handled by the competitor methods are summarized in Table 5 in the supplemental material.

### 5.3 Experimental Setting

For baseline implementations, we either leverage OpenHGNN<sup>6</sup> or use the released source code for evaluation. In our experiments, we keep  $d=200$  for all compared methods. Others hyperparameter settings are considered according to their original papers. For our MHGCN, we set the number of convolution layers  $l$  to 2. For fair comparison, we uniformly set the number of training rounds to 500 for link prediction and the number of training rounds to 200 for node classification. More detailed experimental settings can be found in the supplementary material.

### 5.4 Link Prediction

We first evaluate the model performance by comparing our MHGCN with fifteen baselines on link prediction task in an unsupervised learning manner. The results are shown in Table 2, where the best

<sup>1</sup><https://tianchi.aliyun.com/competition/entrance/231719/information/>

<sup>2</sup><http://jmcauley.ucsd.edu/data/amazon/>

<sup>3</sup><https://github.com/librahu/>

<sup>4</sup>[https://github.com/seongjunyun/Graph\\_Transformer\\_Networks](https://github.com/seongjunyun/Graph_Transformer_Networks)

<sup>5</sup>[https://www.dropbox.com/s/yh4grpeks87ugr2/DBLP\\_processed.zip?dl=0](https://www.dropbox.com/s/yh4grpeks87ugr2/DBLP_processed.zip?dl=0)

<sup>6</sup><https://github.com/BUPT-GAMMA/OpenHGNN>

**Table 2: Model performance comparison for the task of link prediction on different datasets.**

Method	AMiner			Alibaba			IMDB			Amazon			DBLP		
	R-AUC	PR-AUC	F1	R-AUC	PR-AUC	F1	R-AUC	PR-AUC	F1	R-AUC	PR-AUC	F1	R-AUC	PR-AUC	F1
node2vec	0.594	0.663	0.602	0.614	0.580	0.593	0.479	0.568	0.474	0.946	0.944	0.880	0.449	0.452	0.478
RandNE	0.607	0.630	0.608	0.877	0.888	0.826	0.901	0.933	0.839	0.950	0.941	0.903	0.492	0.491	0.493
FastRP	0.620	0.634	0.600	0.927	0.900	0.926	0.869	0.893	0.811	0.954	0.945	0.893	0.515	0.528	0.506
SGC	0.589	0.585	0.567	0.686	0.708	0.623	0.826	0.889	0.769	0.791	0.802	0.760	0.601	0.606	0.587
R-GCN	0.599	0.601	0.610	0.674	0.710	0.629	0.826	0.878	0.790	0.811	0.820	0.783	0.589	0.592	0.566
MAGNN	0.663	0.681	0.666	0.961	0.963	0.948	0.912	0.923	0.887	0.958	0.949	0.915	0.690	0.699	0.684
HPN	0.658	0.664	0.660	0.958	0.961	0.950	0.900	0.903	0.892	0.949	0.949	0.904	0.692	0.710	0.687
PMNE-n	0.651	0.669	0.677	0.966	0.973	0.891	0.674	0.683	0.646	0.956	0.945	0.893	0.672	0.679	0.663
PMNE-r	0.615	0.653	0.662	0.859	0.915	0.824	0.646	0.646	0.613	0.884	0.890	0.796	0.637	0.640	0.629
PMNE-c	0.613	0.635	0.657	0.597	0.591	0.664	0.651	0.634	0.630	0.934	0.934	0.868	0.622	0.625	0.609
MNE	0.660	0.672	0.681	0.944	0.946	0.901	0.688	0.701	0.681	0.941	0.943	0.912	0.657	0.660	0.635
GATNE	OOT	OOT	OOT	0.981	0.986	0.952	0.872	0.878	0.791	0.963	0.948	0.914	OOT	OOT	OOT
DMGI	OOM	OOM	OOM	0.857	0.781	0.784	0.926	0.935	0.873	0.905	0.878	0.847	0.610	0.615	0.601
FAME	0.687	0.747	0.726	0.993	0.996	0.979	0.944	0.959	0.897	0.959	0.950	0.900	0.642	0.650	0.633
DualHGNN	/	/	/	0.974	0.977	0.966	/	/	/	/	/	/	/	/	/
<b>MHGCN</b>	<b>0.711</b>	<b>0.753</b>	<b>0.730</b>	<b>0.997</b>	<b>0.997</b>	<b>0.992</b>	<b>0.967</b>	<b>0.966</b>	<b>0.959</b>	<b>0.972</b>	<b>0.974</b>	<b>0.961</b>	<b>0.718</b>	<b>0.722</b>	<b>0.703</b>

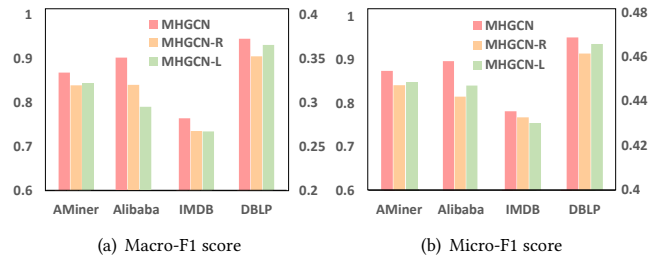
OOT: Out Of Time (36 hours). OOM: Out Of Memory; DMGI runs out of memory on the entire AMiner data. R-AUC: ROC-AUC.

is shown in bold. The first seven baselines are homogeneous or heterogeneous network embedding methods, and the last eight are multiplex network embedding methods.

We can see that MHGCN significantly outperforms all baselines in terms of all evaluation metrics on five datasets. Specifically, MHGCN achieves average gains of 5.68% F1 score in comparison to the best performed GNN baselines across all datasets (*i.e.*, FAME, MAGNN and HPN). Our MHGCN realizes a high accuracy of more than 96% on three datasets (Alibaba, Amazon, and IMDB), especially more than 99% prediction performance on Alibaba network. This is because MHGCN automatically captures effective multi-relational topological structures through multiplex relation aggregation and multilayer graph convolution on the generated meta-paths across multiplex relations. Especially, compared with GATNE and MAGNN, our model has achieved better results, showing the ability of our model in automatically capturing meta-paths compared with manually setting meta-paths. FAME that use spectral graph transformation achieving the second best performance on most datasets also verifies the ability of multiplex relation aggregation to automatically capture useful heterogeneous meta-paths. However, MHGCN obtains better performance than FAME on all networks as MHGCN learns meaning node representations for AMHENS using multilayer graph convolution in a learning manner. Additionally, MHGCN also shows significant performance advantages on general heterogeneous networks (*e.g.*, IMDB and DBLP). This may be because our MHGCN uses a weighted approach to differentiate the effects of different types of relations on node representation, which cannot be achieved by traditional meta-path sampling.

### 5.5 Node Classification

We next evaluate the effectiveness of our model on the node classification task compared with state-of-the-art methods. The results are shown in Table 3, where the best is shown in bold. The first



**Figure 3: Experimental results of ablation study**

eight baselines are unsupervised embedding methods, and the rest are semi-supervised embedding methods.

As we see, MHGCN also achieves state-of-the-art performance on all tested networks. Specifically, our MHGCN achieves average 11.22% and 14.49% improvement over state-of-the-art GNN model HGSL across all datasets in terms of Macro-F1 and Micro-F1, respectively. Considering that the performance gain in node classification task reported in some recent works [5, 43] is usually around 2-4%, this performance improvement achieved by our MHGCN is significant. Furthermore, we also observe that MHGCN performs much better than competitor methods on general heterogeneous network with multi-typed nodes (*e.g.*, IMDB and AMiner), achieving 23.23% and 22.19% improvement in Macro-F1 and Micro-F1 on IMDB network. The possible reason is that our MHGCN effectively learns node representations for classification by exploring all meta-path interactions across multiple relations with different importance (*i.e.*, weights), which is ignored by the heterogeneous network embedding approaches based on manually setting meta-path sampling.

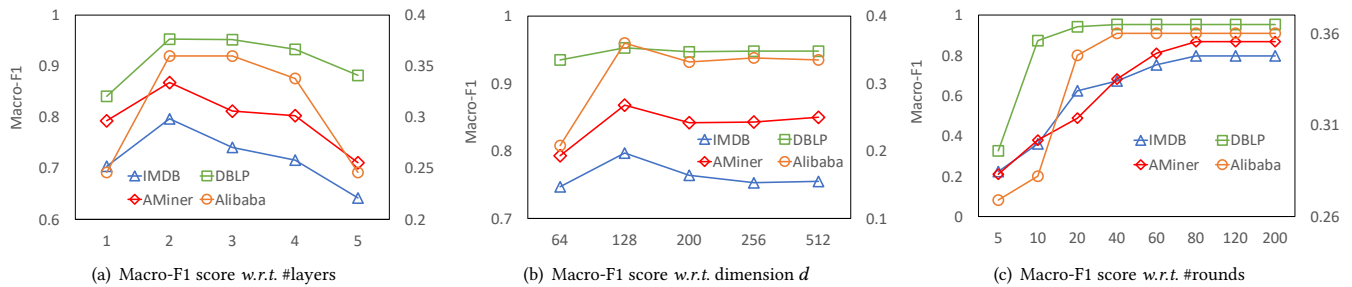
### 5.6 Ablation Study

To validate the effectiveness of each component of our model, we further conduct experiments on different MHGCN variations. Here

**Table 3: Node classification performance comparison of different methods on four datasets**

Method	AMiner		Alibaba		IMDB		DBLP	
	Macro-F1	Micro-F1	Macro-F1	Micro-F1	Macro-F1	Micro-F1	Macro-F1	Micro-F1
node2vec	0.522 (0.0032)	0.532 (0.0051)	0.238 (0.0125)	0.347 (0.0093)	0.363 (0.0237)	0.382 (0.0703)	0.352 (0.0103)	0.351 (0.0112)
RandNE	0.641 (0.0074)	0.672 (0.0064)	0.319 (0.0170)	0.358 (0.0093)	0.373 (0.0143)	0.392 (0.0185)	0.351 (0.0153)	0.372 (0.0150)
FastRP	0.650 (0.0086)	0.690 (0.0074)	0.301 (0.0180)	0.392 (0.0119)	0.363 (0.0236)	0.381 (0.0140)	0.343 (0.0201)	0.375 (0.0199)
MNE	0.643 (0.0069)	0.686 (0.0045)	0.289 (0.0155)	0.390 (0.0021)	0.374 (0.0153)	0.382 (0.0680)	0.366 (0.0117)	0.384 (0.0109)
GATNE	OOT	OOT	0.291 (0.0086)	0.390 (0.0014)	0.369 (0.0132)	0.333 (0.0005)	OOT	OOT
DMGI	0.473 (0.0155)	0.626 (0.0093)	0.220 (0.0214)	0.392 (0.0026)	0.548 (0.0190)	0.544 (0.0189)	0.781 (0.0303)	0.787 (0.0235)
FAME	0.722 (0.0114)	0.727 (0.0091)	0.323 (0.0154)	0.393 (0.0060)	0.593 (0.0135)	0.594 (0.0143)	0.842 (0.0183)	0.868 (0.0127)
DualHGNN	/	/	0.347 (0.0114)	0.402 (0.0127)	/	/	/	/
SGC	0.516 (0.0047)	0.587 (0.0157)	0.286 (0.0231)	0.361 (0.0175)	0.489 (0.0106)	0.563 (0.0133)	0.622 (0.0009)	0.623 (0.0009)
AM-GCN	0.702 (0.0175)	0.713 (0.0223)	0.307 (0.0232)	0.399 (0.0156)	0.610 (0.0021)	0.640 (0.0013)	0.867 (0.0105)	0.878 (0.0112)
R-GCN	0.690 (0.0078)	0.692 (0.0106)	0.265 (0.0326)	0.381 (0.0125)	0.544 (0.0172)	0.572 (0.0145)	0.862 (0.0053)	0.870 (0.0070)
HAN	0.690 (0.0149)	0.726 (0.0086)	0.275 (0.0327)	0.392 (0.0081)	0.552 (0.0112)	0.568 (0.0078)	0.806 (0.0078)	0.813 (0.0100)
NARS	0.722 (0.0103)	0.721 (0.0097)	0.297 (0.0201)	0.392 (0.0195)	0.565 (0.0037)	0.574 (0.0048)	0.794 (0.0255)	0.804 (0.0320)
MAGNN	0.755 (0.0105)	0.757 (0.0133)	0.348 (0.0488)	0.398 (0.0405)	0.614 (0.0073)	0.615 (0.0089)	0.881 (0.0284)	0.895 (0.0396)
HPN	0.710 (0.0612)	0.732 (0.0490)	0.263 (0.0346)	0.392 (0.0405)	0.578 (0.0023)	0.584 (0.0021)	0.822 (0.0201)	0.830 (0.0201)
GTN	OOM	OOM	0.255 (0.0420)	0.392 (0.0071)	0.615 (0.0108)	0.616 (0.0093)	0.852 (0.0137)	0.868 (0.0125)
HGSL	0.754 (0.0100)	0.758 (0.0103)	0.338 (0.0121)	0.398 (0.0238)	0.620 (0.0048)	0.638 (0.0030)	0.893 (0.0284)	0.902 (0.0396)
<b>MHGCN</b>	<b>0.868 (0.0160)</b>	<b>0.875 (0.0200)</b>	<b>0.351 (0.0204)</b>	<b>0.458 (0.0160)</b>	<b>0.764 (0.0145)</b>	<b>0.782 (0.0138)</b>	<b>0.945 (0.0221)</b>	<b>0.952 (0.0203)</b>

OOT: Out Of Time (36 hours), OOM: Out Of Memory. The standard deviations are reported in the parentheses.

**Figure 4: Hyperparameter impact study of the proposed method w.r.t. #layers, dimension  $d$ , and #rounds.**

MHGCN-R does not consider the importance of different relations, that is, we set the weights  $\beta_r$  to 1; MHGCN-L uses only a two-layer GCN to obtain the embedding, so it can only capture the 2-length meta-paths. We report the results of ablation study on four datasets for node classification in Figure 3, where the performance on Alibaba refers to the right-ordinate axis.

It can be seen from the results that the two key components both contribute to performance improvement of our MHGCN. The comparison between MHGCN-R and MHGCN highlights the effectiveness of the importance of different relations. We can observe that MHGCN-R performs worse than MHGCN on all datasets in terms of both Macro-F1 and Micro-F1 metrics, reducing 9.68% performance in Macro-F1 score on Alibaba, which demonstrates the crucial role of our designed multiplex relation aggregation module in capturing the importance of different relations for node representation learning. The comparison between MHGCN-L and MHGCN reflects the importance of our multilayer graph convolution module. Compared with MHGCN-L, MHGCN improves 2.97%, 18.98%, 4.09% and 1.51% over MHGCN-L in terms of Macro-F1 on AMiner, Alibaba,

IMDB, and DBLP, respectively. This indicates that our proposed multilayer graph convolution module effectively captures useful meta-paths of different lengths across multiplex relations.

## 5.7 Parameter Sensitivity

We conduct hyperparameter study in our new MHGCN in terms of key parameters, *i.e.* the number of graph layers  $l$ , the hidden dimension size of embeddings  $d$ , and the number of training rounds. We report Macro-F1 score on node classification task with different parameter settings on four datasets in Figure 4. Notice that the performance on Alibaba refers to the ordinate on the right.

From the results shown in Figure 4(a), we can observe that the best performance can be achieved with two layers. This observation indicates that considering the meta-path interactions with two-hops is sufficient to capture the node dependencies in the graph. Performing the message passing across more graph layers may involve noisy information for node representation. With the growth of GCN layers, the representation of nodes would be flattened after multiple convolutions, resulting in performance degradation. Additionally,



we can notice that the increasing of embedding dimension first brings benefits for the performance improvement, and then leads to the performance degradation. The best prediction accuracy can be achieved with the setting of  $d = 128$ . This is because the features of all nodes are compressed into a small embedding space when dimension  $d$  is small, thus it is difficult to retain the characteristics proximities of all node pairs. Conversely, a larger dimension would also flatten the distance between all node embeddings. Figure 4(c) illustrates the performance of our MHGCN with respect to the number of training rounds in learning model weights. We can find that our MHGCN can converge quickly and efficiently achieve stable performance within 80 rounds on all tested datasets.

## 6 CONCLUSION

In this paper, we propose an embedding model MHGCN for attributed multiplex heterogeneous networks. Our model mainly includes two key components: multiplex relation aggregation and multilayer graph convolution module. Through multiplex relation aggregation, MHGCN can distinguish the importance of the relations between different nodes in multiplex heterogeneous networks. Through multilayer graph convolution module, MHGCN can automatically capture the short and long meta-path interactions across multi-relations, and learn meaning node embeddings with model parameter learning during training phase. Experiments results on five real-world heterogeneous networks show the superiority of the proposed MHGCN in different graph representation tasks.

## ACKNOWLEDGMENTS

This work is partially supported by the National Natural Science Foundation of China under grant Nos. 62176243, 62072288, 61773331 and 41927805, and the National Key Research and Development Program of China under grant Nos. 2018AAA0100602 and 2019YFC1509100.

## REFERENCES

- [1] Yukuo Cen, Xu Zou, Jianwei Zhang, Hongxia Yang, Jingren Zhou, and Jie Tang. 2019. Representation Learning for Attributed Multiplex Heterogeneous Network. In *KDD*. 1358–1368.
- [2] Haochen Chen, Syed Fahad Sultan, Yingtao Tian, Muhao Chen, and Steven Skiena. 2019. Fast and Accurate Network Embeddings via Very Sparse Random Projection. In *CIKM*. 399–408.
- [3] Hongxu Chen, Hongzhi Yin, Weiqing Wang, Hao Wang, Quoc Viet Hung Nguyen, and Xue Li. 2018. PME: projected metric embedding on heterogeneous networks for link prediction. In *KDD*. 1177–1186.
- [4] Yuxiao Dong, Nitesh V Chawla, and Ananthram Swami. 2017. metapath2vec: Scalable representation learning for heterogeneous networks. In *KDD*. 135–144.
- [5] Xinyu Fu, Jiani Zhang, Ziqiao Meng, and Irwin King. 2020. Magnn: Metapath aggregated graph neural network for heterogeneous graph embedding. In *WWW*. 2331–2341.
- [6] Aditya Grover and Jure Leskovec. 2016. node2vec: Scalable feature learning for networks. In *KDD*. ACM, 855–864.
- [7] William L. Hamilton, Rex Ying, and Jure Leskovec. 2017. Inductive Representation Learning on Large Graphs. In *NeurIPS*. 1025–1035.
- [8] Li He, Hongxu Chen, Dingxian Wang, Shoaib Jameel, Philip Yu, and Guandong Xu. 2021. Click-Through Rate Prediction with Multi-Modal Hypergraphs. In *CIKM*. 690–699.
- [9] Xiangnan He, Kuan Deng, Xiang Wang, Yan Li, Yongdong Zhang, and Meng Wang. 2020. Lightgcn: Simplifying and powering graph convolution network for recommendation. In *SIGIR*. 639–648.
- [10] Binbin Hu, Yuan Fang, and Chuan Shi. 2019. Adversarial Learning on Heterogeneous Information Networks. In *KDD*. 120–129.
- [11] Ziniu Hu, Yuxiao Dong, Kuansan Wang, and Yizhou Sun. 2020. Heterogeneous graph transformer. In *WWW*. 2704–2710.
- [12] Chao Huang. 2021. Recent advances in heterogeneous relation learning for recommendation. *arXiv preprint arXiv:2110.03455* (2021).
- [13] Chao Huang, Huance Xu, Yong Xu, Peng Dai, Lianghao Xiao, Mengyin Lu, Liefeng Bo, Hao Xing, Xiaoping Lai, and Yanfang Ye. 2021. Knowledge-aware coupled graph neural network for social recommendation. In *AAAI*.
- [14] Houye Ji, Xiao Wang, Chuan Shi, Bai Wang, and Philip Yu. 2021. Heterogeneous Graph Propagation Network. *TKDE* (2021).
- [15] Baoyu Jing, Chanyoung Park, and Hanghang Tong. 2021. Hdmi: High-order deep multiplex infomax. In *The Web Conference*. 2414–2424.
- [16] Thomas N. Kipf and Max Welling. 2017. Semi-Supervised Classification with Graph Convolutional Networks. In *ICLR*.
- [17] Zhao Li, Xin Shen, Yuhang Jiao, Xuming Pan, Pengcheng Zou, Xianling Meng, Chengwei Yao, and Jiajun Bu. 2020. Hierarchical bipartite graph neural networks: Towards large-scale e-commerce applications. In *ICDE*. IEEE, 1677–1688.
- [18] Weiyei Liu, Pin-Yu Chen, Sailung Yeung, Toyotaro Suzumura, and Lingli Chen. 2017. Principled multilayer network embedding. In *ICDMW*. IEEE, 134–141.
- [19] Zhijun Liu, Chao Huang, Yanwei Yu, and Junyu Dong. 2021. Motif-preserving dynamic attributed network embedding. In *WWW*. 1629–1638.
- [20] Zhijun Liu, Chao Huang, Yanwei Yu, Baode Fan, and Junyu Dong. 2020. Fast Attributed Multiplex Heterogeneous Network Embedding. In *CIKM*. 995–1004.
- [21] Xiaoling Long, Chao Huang, Yong Xu, Huance Xu, Peng Dai, Lianghao Xia, and Liefeng Bo. 2021. Social Recommendation with Self-Supervised Metagraph Informax Network. In *CIKM*. 1160–1169.
- [22] Yuanfu Lu, Chuan Shi, Linmei Hu, and Zhiyuan Liu. 2019. Relation structure-aware heterogeneous information network embedding. In *AAAI*. 4456–4463.
- [23] Chanyoung Park, Donghyun Kim, Jiawei Han, and Hwanjo Yu. 2020. Unsupervised Attributed Multiplex Network Embedding. In *AAAI*. 5371–5378.
- [24] Bryan Perozzi, Rami Al-Rfou, and Steven Skiena. 2014. DeepWalk: online learning of social representations. In *KDD*. 701–710.
- [25] Jiezhong Qiu, Yuxiao Dong, Hao Ma, Jian Li, Chi Wang, Kuansan Wang, and Jie Tang. 2019. Netsmf: Large-scale network embedding as sparse matrix factorization. In *WWW*. 1509–1520.
- [26] Michael Schlichtkrull, Thomas N Kipf, Peter Bloem, Rianne Van Den Berg, Ivan Titov, and Max Welling. 2018. Modeling relational data with graph convolutional networks. In *European semantic web conference*. Springer, 593–607.
- [27] Chuan Shi, Binbin Hu, Wayne Xin Zhao, and S Yu Philip. 2018. Heterogeneous information network embedding for recommendation. *TKDE* 31, 2 (2018), 357–370.
- [28] Jian Tang, Meng Qu, Mingzhe Wang, Ming Zhang, Jun Yan, and Qiaozhu Mei. 2015. Line: Large-scale information network embedding. In *WWW*. 1067–1077.
- [29] Petar Velickovic, Guillem Cucurull, Arantxa Casanova, Adriana Romero, Pietro Liò, and Yoshua Bengio. 2018. Graph Attention Networks. In *ICLR*.
- [30] Xiao Wang, Houye Ji, Chuan Shi, Bai Wang, Yanfang Ye, Peng Cui, and Philip S Yu. 2019. Heterogeneous Graph Attention Network. In *WWW*. ACM, 2022–2032.
- [31] Xiao Wang, Yuanfu Lu, Chuan Shi, Ruijia Wang, Peng Cui, and Shuai Mou. 2020. Dynamic heterogeneous information network embedding with meta-path based proximity. *TKDE* (2020).
- [32] Xiao Wang, Meiqi Zhu, Deyu Bo, Peng Cui, Chuan Shi, and Jian Pei. 2020. Am-gcn: Adaptive multi-channel graph convolutional networks. In *KDD*. 1243–1253.
- [33] Wei Wei, Chao Huang, Lianghao Xia, Yong Xu, Jiashu Zhao, and Dawei Yin. 2022. Contrastive meta learning with behavior multiplicity for recommendation. In *KDD*. 1120–1128.
- [34] Felix Wu, Amauri Souza, Tianyi Zhang, Christopher Fifty, Tao Yu, et al. 2019. Simplifying Graph Convolutional Networks. In *ICML*. 6861–6871.
- [35] Yongji Wu, Defu Lian, Yiheng Xu, Le Wu, and Enhong Chen. 2020. Graph convolutional networks with markov random field reasoning for social spammer detection. In *AAAI*, Vol. 34. 1054–1061.
- [36] Lianghao Xia, Chao Huang, Yong Xu, Peng Dai, Bo Zhang, and Liefeng Bo. 2020. Multiplex Behavioral Relation Learning for Recommendation via Memory Augmented Transformer Network. In *SIGIR*. 2397–2406.
- [37] Hansheng Xue, Luwei Yang, Vaibhav Rajan, Wen Jiang, Yi Wei, and Yu Lin. 2021. Multiplex bipartite network embedding using dual hypergraph convolutional networks. In *WWW*. 1649–1660.
- [38] Lingfan Yu, Jiajun Shen, Jinyang Li, and Adam Lerer. 2020. Scalable graph neural networks for heterogeneous graphs. *arXiv preprint arXiv:2011.09679* (2020).
- [39] Seongjun Yun, Minbyul Jeong, Raehyun Kim, Jaewoo Kang, and Hyunwoo J. Kim. 2019. Graph Transformer Networks. In *NeurIPS*. 11960–11970.
- [40] Chuxu Zhang, Dongjin Song, Chao Huang, Ananthram Swami, and Nitesh V Chawla. 2019. Heterogeneous graph neural network. In *KDD*. 793–803.
- [41] Hongming Zhang, Liwei Qiu, Lingling Yi, and Yangqiu Song. 2018. Scalable multiplex network embedding. In *IJCAI*, Vol. 18. 3082–3088.
- [42] Ziwei Zhang, Peng Cui, Haoyang Li, Xiao Wang, and Wenwu Zhu. 2018. Billion-Scale Network Embedding with Iterative Random Projection. In *ICDM*. 787–796.
- [43] Jianan Zhao, Xiao Wang, Chuan Shi, Binbin Hu, Guojie Song, and Yanfang Ye. 2021. Heterogeneous Graph Structure Learning for Graph Neural Networks. In *AAAI*.

## A SUPPLEMENTAL MATERIAL

### A.1 Notations

We summarize the key notations used in the paper as well as their definitions in Table 4.

**Table 4: Summary of key notations**

Notation	Definition
$\mathcal{G}$	The target graph
$\mathcal{V}, \mathcal{E}$	the set of nodes and edges in $\mathcal{G}$
$\mathcal{O}, \mathcal{R}$	the set of node and edge types in $\mathcal{G}$
$\mathbf{X}$	the matrix of node attributes in $\mathcal{G}$
$\mathcal{G}_r$	the sub-network w.r.t. edge type $r$
$\mathbf{A}_r$	the adjacency matrix of $\mathcal{G}_r$
$\mathbf{A}$	the aggregated adjacency matrix
$\mathbf{H}$	the node embeddings
$\mathbf{H}^{(l)}$	the hidden representation for the $l$ -th layer
$d$	the hidden dimensionality of embeddings
$n, m$	the number of nodes and attributes
$\beta_r$	the learnable weight for edge type $r$
$\mathbf{W}^{(l)}$	the learnable weight matrix for the $l$ -th layer

### A.2 Algorithm Pseudo-Code

Algorithm 1 shows the pseudo-code of our proposed MHGCN framework guided by the above objective functions (*i.e.*, Eq. (6) or Eq. (7)).

---

**Algorithm 1** The Learning Procedure of our MHGCN Model

---

**Input:** The generated AMHEN  $\mathcal{G}$  and node feature matrix  $\mathbf{X}$ .

**Output:** The node embeddings  $\mathbf{H}$  of graph  $\mathcal{G}$ .

- 1: We generate the adjacency matrices  $\{\mathbf{A}_r | r = 1, 2, \dots, |\mathcal{R}|\}$  by decoupling the attributed multiplex heterogeneous network into homogeneous and bipartite graphs.
  - 2: Calculate  $\mathbf{A} = \sum_{r=1}^{|\mathcal{R}|} \beta_r \mathbf{A}_r$
  - 3: **for**  $i = 1$  to  $l$  **do**
  - 4:   Calculate  $\mathbf{H}^{(i)} \leftarrow \mathbf{A} \cdot \mathbf{H}^{(i-1)} \cdot \mathbf{W}^{(i)}$
  - 5: **end for**
  - 6:  $\mathbf{H} = \frac{1}{l} (\mathbf{H}^{(1)} + \dots + \mathbf{H}^{(l)})$
  - 7: Calculate  $\mathcal{L}$  using Eq. (6) or Eq. (7);
  - 8: Back propagation and update parameters in MHGCN
  - 9: Return  $\mathbf{H}$
- 

### A.3 Detailed Dataset Description

i) For Alibaba dataset, four types of user-item interactions are regarded as the node-wise multiplex relationships. The item categories are considered as the ground truth labels for node classification. ii) For Amazon dataset, the multiplex edges are represented as the co-viewing and co-purchasing relations between different products. The node attributes include the external features of products, *e.g.*, category, sales-rank, brand and price information. iii) For the AMiner dataset, three types of nodes (*i.e.*, author, paper and conference) are included in the heterogeneous graph. The node labels are the paper domains. iv) For the IMDB dataset, movie, director and

actor construct the heterogeneous nodes. We consider the genres of movies as the node labels. The bag-of-words representations are considered as the node attributed feature vectors.

v) For the DBLP dataset, four types of nodes are involved in the heterogeneous graph, namely, author, paper, venue, and term. We regard the research field of authors as the node class labels.

### A.4 Detailed Experimental Settings

For link prediction task, we treat the connected nodes in network as positive node pairs, and consider all unlinked nodes as negative node pairs. For each edge type, we divide the positive node pairs into training set, verification set and test set according to the proportion of 85%, 5% and 10%. At the same time, we randomly select the same number of negative node pairs to add into training set, validation set and test set. Notice that we predict each type of edge using all types of edges in datasets, and finally take the average of all edges as the final result. In particular, the training, validation and test sets are generated with the ratio of 80%, 10% and 10%, respectively. In experiments, we train a logistic regression classifier for node classification. Notice that we repeat each experiment 10 times to report average results.

For fair comparison, we uniformly set the number of training rounds to 500 for link prediction and the number of training rounds to 200 for node classification. For node2vec method, the parameters  $p$  and  $q$  for random walk control are set as 2 and 0.5, respectively. For GATNE approach, the parameters  $\alpha_r$  and  $\beta_r$  are set as 1 for each edge type. For the MNE, we set the dimension of additional vectors to 10, set the length of walk as 10, set the number of walks as 20. For compared neural network-based models, the learning rate is searched from the range of  $\{0.0005, 0.0001, 0.005, 0.001, 0.05, 0.01, \dots\}$ . We configure the multi-head attention with 8 head-specific representation spaces, and apply the dropout ratio of 0.6. For GTN method, the number of graph Transformer layers is set as 3. For DMGI baseline, the parameters  $\alpha, \beta, \gamma$  are chosen from the value range of  $\{0.0001, 0.001, 0.01, 0.1\}$ . The weight  $w$  for self-connection is set as 3.

For FAME, we use optuna<sup>7</sup> to tune the parameters over  $\alpha_1, \dots, \alpha_K$  and  $\beta_1, \dots, \beta_{|\mathcal{R}|}$  as described in the original paper. For AM-GCN, we tune loss aggregation parameters  $\beta, \gamma$  in  $\{0.0001, 0.001, 0.01, 0.1\}$ . For MAGNN, we set the number of independent attention mechanisms  $k = 4$ . For HPN, we set iterations in semantic propagation  $k = 3$  and value of restart probability  $\alpha = 0.1$ . For HGSL, we set the number of GNN layers to 2 and the hidden layer output dimension to 64. For R-GCN, we set the batch size to 126, the number of GNN layers to 2, and the hidden layer dimension to 64. For NARS, we set the number of hops to 2, and the number of feed-forward layers to 2. For DualHGNN, we use the asymmetric operator and set  $\lambda$  as 0.5.

For our MHGCN, we set the number of convolution layers  $l$  to 2, learning rate to 0.05, dropout to 0.5, and weight-decay to 0.0005.

The configurations of system platform for efficiency evaluation are as followed. CPU: Intel Xeon E5-2660 (2.2GHz), Memory: 80GB, 2 GPU units: GeForce RTX 2080 (8G).

The source code of our model implementation is available at <https://github.com/NSSSJSS/MHGCN>.

<sup>7</sup><https://github.com/pfnet/optuna>

**Table 5: The types of graphs handled by different methods (Heter.: Heterogenous node and edge types, Multi.: Multiplex edges, Attr.: Node attributed information, Unsup.: Unsupervised learning, Auto.: Automatic meta-path).**

Method	Heter.		Multi.	Attr.	Unsup.	Auto.
	Node	Edge				
node2vec	×	×	×	×	✓	×
RandNE	×	×	×	×	✓	×
FastRP	×	×	×	×	✓	×
SGC	×	×	×	✓	✓/×	×
AM-GCN	×	×	×	✓	×	×
R-GCN	✓	✓	×	✓	✓/×	×
HAN	✓	✓	×	✓	×	×
NARS	✓	✓	×	✓	×	×
MAGNN	✓	✓	×	✓	✓/×	×
HPN	✓	✓	×	✓	✓/×	×
PMNE	×	✓	✓	×	✓	×
MNE	×	✓	✓	×	✓	×
GATNE	✓	✓	✓	✓	✓	×
GTN	✓	✓	✓	✓	×	✓
DMGI	✓	✓	✓	✓	✓	×
FAME	✓	✓	✓	✓	✓	✓
HGSL	✓	✓	✓	✓	×	×
DualHGNN	✓	×	✓	✓	✓	×
MHGCN	✓	✓	✓	✓	✓/×	✓

### A.5 Baselines

The publicly source codes of baselines can be available at the following URLs:

- **node2vec** – <https://github.com/aditya-grover/node2vec>
- **RandNE** – <https://github.com/ZW-ZHANG/RandNE>
- **FastRP** – <https://github.com/GTmac/FastRP>
- **SGC** – <https://github.com/Tiiiger/SGC>
- **AM-GCN** – <https://github.com/zhumeiqiBUPT/AM-GCN>
- **R-GCN** – <https://github.com/BUPT-GAMMA/OpenHGNN>
- **HAN** – <https://github.com/Jhy1993/HAN>
- **NARS** – <https://github.com/BUPT-GAMMA/OpenHGNN>
- **MAGNN** – <https://github.com/cynricfu/MAGNN>
- **HPN** – <https://github.com/BUPT-GAMMA/OpenHGNN>
- **PMNE** – The source code of PMNE used in this work is released by the authors of MNE at <https://github.com/HKUST-KnowComp/MNE>
- **MNE** – <https://github.com/HKUST-KnowComp/MNE>
- **GATNE** – <https://github.com/THUDM/GATNE>
- **GTN** – [https://github.com/seongjunyun/Graph\\_Transformer\\_Networks](https://github.com/seongjunyun/Graph_Transformer_Networks)
- **DMGI** – <https://github.com/pcy1302/DMGI>
- **FAME** – <https://github.com/ZhijunLiu95/FAME>
- **HGSL** – <https://github.com/Andy-Border/HGSL>
- **DualHGNN** – <https://github.com/xuehansheng/DualHGNN>

For homogeneous network embedding methods and heterogeneous network embedding methods to deal with multiplex networks, we feed separate graphs with a single-layer view into them

to obtain different node embeddings, then perform mean pooling to generate final node embedding. Since DualHGNN is designed only for multiplex bipartite networks, it can only work on Alibaba network.

The network types handled by the baseline methods are summarized in Table 5.

### A.6 Additional Experimental Results

**A.6.1 Model Efficiency Analysis.** We also compare the efficiency of our MHGNN with other GNN baselines for semi-supervised node classification. We report the experimental results on four datasets in Table 6.

As can be seen from Table 6, our MHGNN achieves the fourth-best performance after three heterogeneous network embedding methods (*i.e.*, R-GCN, NARS and HPN). However, from the above experimental results (Tables 2 and 3), MHGNN is significantly better than these three methods in both link prediction and node classification. MHGNN is significantly faster than the best performed GNN baseline in node classification task (*i.e.*, HGSL) on all datasets under the same number of training rounds. More specifically, our MHGNN achieves up to 135× speedup over state-of-the-art embedding method HAN. MHGNN is faster than state-of-the-art AMHEN embedding method GTN by 21.25 times on multiplex Alibaba network. MHGNN is even 2.33 times and 16.58 times faster than state-of-the-art heterogeneous GNN model MAGNN on Alibaba and AMiner, respectively. The main reason is because our MHGNN adopts the idea of simplifying graph convolutional networks, that is, omitting non-linear activation function. Therefore, the training efficiency of MHGNN can be significantly improved. In fact, according to the above experimental results in Figure 4(c), our model can converge quickly within 80 rounds for node classification on four tested datasets, that is, our model does not need to be trained for 200 rounds set in our experimental evaluation and thus can achieve faster efficiency.

**Table 6: Runtime comparison of GNN methods (Second)**

Method	AMiner	Alibaba	IMDB	DBLP
AM-GCN	8703.71	2519.82	24280.12	2786.73
R-GCN	<b>153.04</b>	301.25	155.40	192.85
HAN	87105.55	4226.95	70510	22315.36
NARS	172.21	<b>211.54</b>	<b>75.81</b>	<b>108.54</b>
MAGNN	10361.20	2320.62	731.03	2125.33
HPN	172.82	249.47	176.64	109.49
GTN	OOM	21166.83	4287.20	18233.64
HGSL	1684.03	2120.93	1758.21	2037.10
DualHGNN	/	11295.92	/	/
<b>MHGNN</b>	645.20	996.52	677.23	970.29
Speedup*	<b>135.05×</b>	<b>4.37×</b>	<b>104.15×</b>	<b>23.01×</b>
Speedup**	/	<b>21.25×</b>	<b>6.33×</b>	<b>18.80×</b>

\* Speedup of MHGNN over HAN.

\*\* Speedup of MHGNN over GTN.

OOM: Out Of Memory.